Analysis of Innovations program documentation

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/home/ross/DataAssim/Meso/AnalofInnovs/Documentation.lyx

User options

For descriptions of the parameters, see below.

Run mode 1: Test reading and writing 1 forecast data file and auxiliary data

```
diamet.out 1 working_dir
```

- The model forecast data filename is in hardwired in the code.

Run mode 2: Test reading and outputting observation values

```
diamet.out 2 working_dir obs_dir num_obs_files \
           obs_list_filename \
           T? u? v? p? q? long_wind_uv? trans_wind_uv? long_wind_radar?
```

Run mode 3: Read forecasts and observations, compute model observations, and output

```
diamet.out 3 working_dir obs_dir num_obs_files \
           obs_list_filename \
           inc_w_in_modelobs? \
           T? u? v? p? q? long_wind_uv? trans_wind_uv? long_wind_radar?
```

- The model forecast data filename is in hardwired in the code.
- The include_w_in_modelob_calc flag (0/1).

Run mode 4 : Read output from run mode 4, compute obs vs model obs, and compute analysis of innovations covariances

```
diamet.out 4 working_dir \
           num_vert_bins top_height \
           num_horiz_bins max_horiz_dist \
           min_fc_lead_time max_fc_lead_time time_window radar_angle_deg \
           T? u? v? p? q? long_wind_uv? trans_wind_uv? long_wind_radar?
```

Descriptions of the parameters

working_dir The directory containing the input data (except observations – see below), and to where the outputs are sent.

obs_dir The directory containing the observation files.

num_obs_files The number of observation files.

obs_list_filename The file (inside working_dir) that specifies the observation files to be read-in (see below for the format of this file).

num_vert_bins The number of vertical bins to consider.

top_height The top height of the top vertical bin (m).

num_horiz_bins The number of horizontal bins to consider.

max_horiz_dist The distance of the last horizontal bin (m).

min_fc_lead_time The minimum forecast lead time considered (s) (e.g. if considering 3-hour forecast errors, this might be 2.5).

max_fc_lead_time The maximum forecast lead time considered (s) (e.g. if considering 3-hour forecast errors, this might be 3.5).

time_window The window within which observations are considered to have common truth (s).

radar_angle_deg Radar angle to decide whether beams of ob pairs align (degrees)

inc_w_in_modelobs Include w in model observation calculation (relevant only for Doppler radar observations) (0 or 1)?

T? Deal with temperature (0 or 1)?

u? Deal with zonal wind (0 or 1)?

v? Deal with meridional wind (0 or 1)?

p? Deal with pressure (0 or 1)?

q? Deal with water vapour mixing ratio (0 or 1)?

long_wind_uv? Deal with longitudinal wind (0 or 1) (along direction between two observations), where the longitudinal wind is computed from the u and v observations?

trans_wind_uv? Deal with transverse wind (0 or 1) (perpendicular to the direction between two observations), where the transverse wind is computed from the u and v observations?

long_wind_radar? Deal with longitudinal wind (0 or 1) (along direction between two observations), where the longitudinal wind is measured from Doppler radar?

Format of the obs_list_filename file

```
obs_type_1  obs_file_1
obs_type_2  obs_file_2
...
```

- obs_type_n is the type of observation for the nth observation file (AMDAR, TEMP, FAAM, RADAR).
- obs_file_n is the filename of the nth observation file (expected to be inside obs_dir).
- The file listing these things is expected to be placed in the working_dir directory.

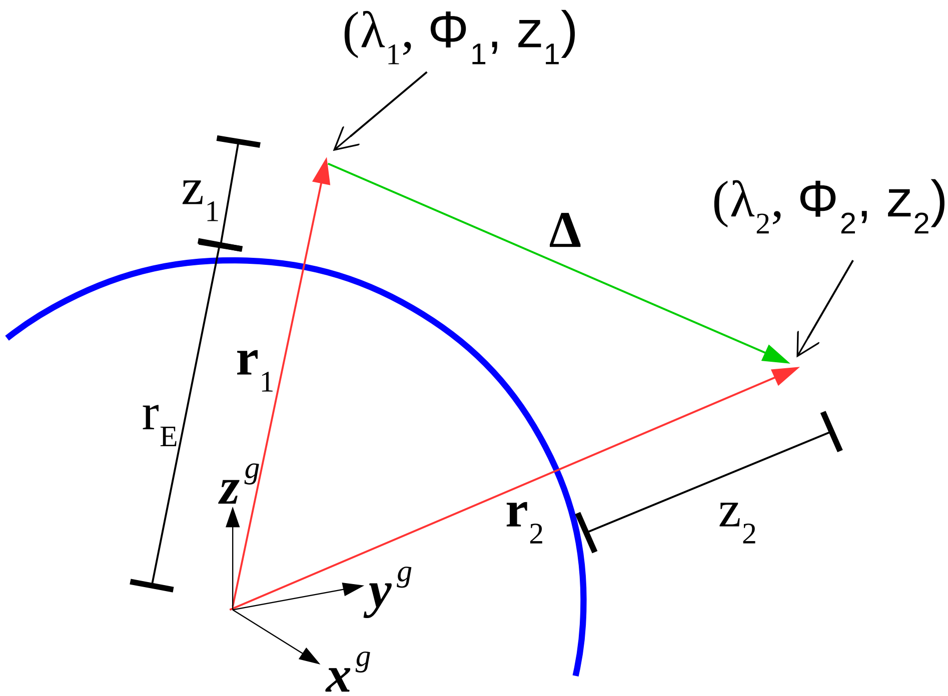

Figure 1: Geometry of two positions on the Earth $\mathbf{r}_1$ and $\mathbf{r}_2$.

Radar geometry formulae

Finding the 'longitude' directions between two points

Consider a point at longitude λ_1, latitude ϕ_1, and height above sea level z_1 (see Fig. 1). The position vector of this point, $\mathbf{r}_1$, in global Cartesian co-ordinates – $(\mathbf{x}^{\mathrm{g}}, \mathbf{y}^{\mathrm{g}}, \mathbf{z}^{\mathrm{g}})$ in Fig. 1 – is

$$\mathbf{r}_1 = \begin{pmatrix} (r_{\mathrm{E}} + z_1)\cos\phi_1\cos\lambda_1 \\ (r_{\mathrm{E}} + z_1)\cos\phi_1\sin\lambda_1 \\ (r_{\mathrm{E}} + z_1)\sin\phi_1 \end{pmatrix}, \tag{1}$$

where r_{E} is the radius of the Earth. The local unit vectors at $\mathbf{r}_1$ forming a local Cartesian co-ordinate system (eastward, northward, upward) are rows of the following transform matrix

$$\mathbf{T}_1 = \begin{pmatrix} -\sin\lambda_1 & \cos\lambda_1 & 0 \\ -\sin\phi_1\cos\lambda_1 & -\sin\phi_1\sin\lambda_1 & \cos\phi_1 \\ \cos\phi_1\cos\lambda_1 & \cos\phi_1\sin\lambda_1 & \sin\phi_1 \end{pmatrix}. \tag{2}$$

Similar formulae exist for point $\mathbf{r}_2$.

To find the unit vector that exists at $\mathbf{r}_1$ and describes the horizontal wind component that is longitudinal between $\mathbf{r}_2$ and $\mathbf{r}_1$, first define the difference (see Fig. 1),

$$\boldsymbol{\Delta} = \mathbf{r}_2 - \mathbf{r}_1, \tag{3}$$

which projects onto the local unit vectors at $\mathbf{r}_1$ as follows

$$\mathbf{v}_1 = \mathbf{T}_1\boldsymbol{\Delta}. \tag{4}$$

We wish to describe a horizontal unit vector linking $\mathbf{r}_1$ and $\mathbf{r}_2$. Eliminating the vertical component of $\mathbf{v}_1$ and normalising

$$\hat{\mathbf{l}}_1 = \mathcal{N}\left[\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}\mathbf{v}_1\right] = \mathcal{N}\left[\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}\mathbf{T}_1\boldsymbol{\Delta}\right], \tag{5}$$

where $\mathcal{N}$ is an instruction to normalise. Given a local wind vector at $\mathbf{r}_1$ (call $\mathbf{u}_1$), the 'longitudinal' component (i.e. the component in the direction of $\mathbf{r}_2$) is then $u_1^{\mathrm{long}} = \hat{\mathbf{l}}_1^{\mathrm{T}}\mathbf{u}_1$. The corresponding longitudinal value at $\mathbf{r}_2$ is $u_2^{\mathrm{long}} = \hat{\mathbf{l}}_2^{\mathrm{T}}\mathbf{u}_2$.

Finding the transverse directions between two points

The transverse direction at $\mathbf{r}_1$ (call $\hat{\mathbf{t}}_1$) is found with the following

$$\hat{\mathbf{t}}_1 = -\hat{\mathbf{l}}_1 \times \hat{\mathbf{z}}_1, \tag{6}$$

where $\hat{\mathbf{z}}_1$ is the local upward unit vector. The transpose component of the wind at $\mathbf{r}_1$ is then $u_1^{\mathrm{trans}} = \hat{\mathbf{t}}_1^{\mathrm{T}}\mathbf{u}_1$, and at $\mathbf{r}_2$ is $u_2^{\mathrm{trans}} = \hat{\mathbf{t}}_2^{\mathrm{T}}\mathbf{u}_2$.

Finding the location of the radar scatterer

Given that $\mathbf{r}_1$ is the location of a radar instrument, which receives a back-scatter from an airmass of specified elevation angle (θ), azimuthal angle (φ, measured clockwise from N), and range (r), what is the longitude, latitude, and height of the airmass (at $\mathbf{r}_2$)?

In the local co-ordinate system of the radar, the position of the airmass is

$$\mathbf{s} = \begin{pmatrix} r\cos\theta\sin\varphi \\ r\cos\theta\cos\varphi \\ r\sin\theta + z_1 \end{pmatrix}. \tag{7}$$

This needs to be converted into global co-ordinates, and added to $\mathbf{r}_1$ to give $\mathbf{r}_2$

$$\mathbf{r}_2 = \mathbf{r}_1 + \mathbf{T}_1^{\mathrm{T}}\mathbf{s}, \tag{8}$$

where $\mathbf{T}_1^{\mathrm{T}}$ is the inverse of the orthogonal matrix $\mathbf{T}_1$ in (2). From $\mathbf{r}_2$ it is possible to then compute λ_2, ϕ_2, and z_2. This uses the version of (1) that applies to $\mathbf{r}_2$ rather than $\mathbf{r}_1$ (in global co-ordinates):

$$x_2^{\mathrm{g}} = (r_{\mathrm{E}} + z_2)\cos\phi_2\cos\lambda_2 \tag{9}$$

$$y_2^{\mathrm{g}} = (r_{\mathrm{E}} + z_2)\cos\phi_2\sin\lambda_2 \tag{10}$$

$$z_2^{\mathrm{g}} = (r_{\mathrm{E}} + z_2)\sin\phi_2. \tag{11}$$

To find λ_2, divide (10) by (9)

$$\frac{y_2^{\mathrm{g}}}{x_2^{\mathrm{g}}} = \tan\lambda_2.$$

This will give an angle $-\pi \leq \lambda_2 < \pi$. If $x_2 < 0$ then π should be added to λ_2 as follows

$$\lambda_2 = \begin{cases} \arctan(y_2^{\mathrm{g}}/x_2^{\mathrm{g}}) & x_2^{\mathrm{g}} > 0 \\ \pi + \arctan(y_2^{\mathrm{g}}/x_2^{\mathrm{g}}) & x_2^{\mathrm{g}} < 0 \\ \pi/2 & x_2^{\mathrm{g}} = 0,\ y_2^{\mathrm{g}} > 0 \\ -\pi/2 & x_2^{\mathrm{g}} = 0,\ y_2^{\mathrm{g}} < 0 \end{cases} \tag{12}$$

Square, then sum (10) and (9)

$$\begin{aligned} x_2^{\mathrm{g}\,2} + y_2^{\mathrm{g}\,2} &= (r_{\mathrm{E}} + z_2)^2\cos^2\phi_2 \\ &= (r_{\mathrm{E}} + z_2)^2(1 - \sin^2\phi_2) \\ \text{so } \sin^2\phi_2 &= 1 - \frac{x_2^{\mathrm{g}\,2} + y_2^{\mathrm{g}\,2}}{(r_{\mathrm{E}} + z_2)^2} \\ &= \frac{(r_{\mathrm{E}} + z_2)^2 - x_2^{\mathrm{g}\,2} - y_2^{\mathrm{g}\,2}}{(r_{\mathrm{E}} + z_2)^2} \\ \text{so } \sin\phi_2 &= \frac{\sqrt{(r_{\mathrm{E}} + z_2)^2 - x_2^{\mathrm{g}\,2} - y_2^{\mathrm{g}\,2}}}{r_{\mathrm{E}} + z_2}. \end{aligned}$$

Combining this result with (11) gives

$$z_2^{\mathrm{g}} = \sqrt{(r_{\mathrm{E}} + z_2)^2 - x_2^{\mathrm{g}\,2} - y_2^{\mathrm{g}\,2}},$$

which can be solved for z_2

$$z_2 = \sqrt{z_2^{\mathrm{g}\,2} + x_2^{\mathrm{g}\,2} + y_2^{\mathrm{g}\,2}} - r_{\mathrm{E}}. \tag{13}$$

The remaining variable is ϕ_2, which comes from (11)

$$\phi_2 = \arcsin \frac{z_2^{\mathrm{g}}}{r_{\mathrm{E}} + z_2}. \tag{14}$$

Finding the projection vector for the radial wind at the scatterer's position

The vector linking $\mathbf{r}_2$ (the position of the scatterer in global co-ordinates) with $\mathbf{r}_1$ (the position of the radar in global co-ordinates) is $\mathbf{T}_1^{\mathrm{T}}\mathbf{s}$ in (8). The projection vector (call $\hat{\mathbf{R}}$) used to find the radial wind at the scatterer's position is $\mathbf{T}_1^{\mathrm{T}}\mathbf{s}$ projected onto the local co-ordinates at $\mathbf{r}_2$, and then normalised for unit length as follows

$$\hat{\mathbf{R}} = \mathcal{N}\left(\mathbf{T}_2\mathbf{T}_1^{\mathrm{T}}\mathbf{s}\right), \tag{15}$$

where $\mathbf{T}_2$ is defined as (2), but for position $\mathbf{r}_2$ instead of $\mathbf{r}_1$.