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Achieving seventh-order amplitude accuracy in leapfrog integrations

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The leapfrog time-stepping scheme makes no amplitude errors when integrating linear oscillations. Unfortunately, the Robert–Asselin filter, which is used to damp the computational mode, introduces first-order amplitude errors. The RAW filter, which was recently proposed as an improvement, eliminates the first-order amplitude errors and yields third-order amplitude accuracy. However, it has not previously been shown how to further improve the accuracy by eliminating the third- and higher-order amplitude errors.

Here, it is shown that leapfrogging over a suitably weighted blend of the filtered and unfiltered tendencies eliminates the third-order amplitude errors and yields fifth-order amplitude accuracy. It is further shown that the use of a more discriminating $(1, -4, 6, -4, 1)$ filter instead of a $(1, -2, 1)$ filter eliminates the fifth-order amplitude errors and yields seventh-order amplitude accuracy. Other related schemes are obtained by varying the values of the filter parameters, and it is found that several combinations offer an appealing compromise of stability and accuracy. The proposed new schemes are tested in numerical integrations of a simple nonlinear system. They appear to be attractive alternatives to the filtered leapfrog schemes currently used in many atmosphere and ocean models.

Reference

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