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Introduction. Stochastic physics and climate modelling

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Finite computing resources limit the spatial resolution of state-of-the-art global climate simulations to hundreds of kilometres. In neither the atmosphere nor the ocean are small-scale processes such as convection, clouds and ocean eddies properly represented. Climate simulations are known to depend, sometimes quite strongly, on the resulting bulk-formula representation of unresolved processes. Stochastic physics schemes within weather and climate models have the potential to represent the dynamical effects of unresolved scales in ways which conventional bulk-formula representations are incapable of so doing. The application of stochastic physics to climate modelling is a rapidly advancing, important and innovative topic. The latest research findings are gathered together in the Theme Issue for which this paper serves as the introduction.

Keywords: climate modelling; stochastic physics; parametrization

1. Introduction and motivation

The dynamical evolution equations for weather and climate are formally deterministic. As such, one might expect that solutions of these dynamical evolution equations are uniquely determined by the imposed initial condition. The purpose of this Theme Issue of *Philosophical Transactions* is to suggest otherwise.

Before expanding on this seemingly paradoxical claim, let us first outline the reason why the theme of this issue is of enormous practical importance. As discussed below, we could legitimately call it a trillion-dollar topic.

While weather forecasting has a long and perhaps chequered history, the present era, whereby predictions are made from numerical solutions of the underlying dynamic and thermodynamic equations, can be traced back to the pioneering work of L. F. Richardson in the early years of the twentieth century (Lynch 2006). Of course, as is well known, the notion that detailed weather forecasts could be made arbitrarily far into the future was dealt a practical blow through the discovery that weather was chaotic, i.e. weather forecasts are sensitive to small errors in their initial conditions (e.g. Lorenz 1993). To some

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One contribution of 12 to a Theme Issue ‘Stochastic physics and climate modelling’.

people, the fact that the weather is chaotic seemed to imply that it is hopeless to try to forecast it. However, a fundamental property of any chaotic system is that the degree to which it is predictable is itself a function of the initial state; forecasts from some initial states can be very predictable, even though the system as a whole is chaotic.

To exploit this property of weather as a chaotic dynamical system, methods based on ensemble forecasting have been developed to try to predict when the weather is predictable and when it is unpredictable. The method is conceptually simple: an ensemble is a collection of forecasts made from almost, but not quite, identical initial conditions. The spread among members of the ensemble gives an estimate of flow-dependent predictability.

In recent years, the ensemble method has become a backbone of numerical weather prediction and is used not only by weather forecasters but also by commercial traders whose activities depend on weather. For example, weather is a dominant driver of many commodities traded in liberalized markets (electricity, gas, coal, oil, crops). Having an estimate of flow-dependent uncertainty in forecasts of weather is critical to the success of such commodity trading, and ensemble weather forecasting is the tool used by the traders to determine this.

Developing practical tools for estimating the uncertainty of a forecast requires a detailed knowledge of the sources of forecast uncertainty. The simple chaotic paradigm discussed above suggests that the only relevant uncertainty lies in the weather observations that determine the initial state of the forecast, e.g. that the measuring instruments are never perfectly accurate or never sufficiently dense in space to determine every small fluctuation in the initial atmospheric state. However, the problem is not nearly as simple as this. Another key source of uncertainty in any weather forecast is the numerical model used to make the predictions.

So let us return to the beginning of this article. The dynamic and thermodynamic equations are given as deterministic partial differential equations, but are solved by discretization onto some sort of grid (or spectral or other equivalent representation). Since there are inevitably scales of motion and indeed key processes that are not resolved by this discretization, methods must be found to represent approximately the subgrid features of the flow. For example, if a global numerical weather prediction problem has a typical grid spacing of 50 km, then all individual cloud systems will be unresolved. For this reason, the numerical equations are 'closed' by adding empirically based subgrid parametrization formulae to represent the effects of the unresolved scales. Hence, for example, convective clouds (e.g. associated with thunderstorms) are represented by convective subgrid parametrization formulae. Other subgrid parametrization formulae represent the effects of flow over and around small-scale topography, boundary-layer turbulence and the absorption and emission of radiation in various relevant parts of the electromagnetic spectrum by radiatively active constituents in our atmosphere.

The formulation of these parametrization formulae is motivated by notions in statistical mechanics. So, just as the momentum transfer by the bulk effects of molecular motions is represented by a diffusive formula, so a similar type of formula might represent the bulk effects of cumulus clouds on vertical temperature, humidity and momentum transfer on the grid scale. However, there is a problem with such an approach. Within a typical 50 km square grid box, there often exist sufficiently few individual cumulus clouds for the parametrized bulk formula to be an accurate estimate of the subgrid effects.

How can we represent this source of error in ensemble forecasts? This is where the concept of stochastic modelling of the subgrid scales is relevant. By representing model uncertainty through stochastic equations (or more generally by stochastic-dynamic models; Palmer 2001), the resulting ensemble forecasts can sample the effects of both initial observation uncertainties and forecast model uncertainties. The resulting ensemble weather forecasts are more reliable (in a precise statistical sense) than those associated with only a sampling of initial observation error, and this has made the whole process of predicting uncertainty more valuable to the real-world customers of weather forecasts.

But this is only half the story! Although weather forecasting has a long history, it is only in recent years that the world has become aware of the threat of climate change. Many regard this as the most serious threat facing humanity—a threat literally to our civilization. Others, while perhaps acknowledging that the world has warmed in recent years and that some of this could be due to man's activities, believe that the climate change problem is not as important as other problems facing society. To some extent, extreme views about climate change, the cataclysmic and the dismissive, arise because there remains considerable uncertainty in the magnitude of future global warming, e.g. as reflected in the Intergovernmental Panel on Climate Change (IPCC) assessment reports. Certainly the IPCC assessment reports show that among the range of model predictions, there is a quantifiable risk of dangerous climate change in the coming century, and most sensible observers deduce from this that the world needs to take action, first to reduce emissions of greenhouse gases and second to start preparing to adapt to inevitable climate change.

Climate-change predictions will play a key role in both mitigation and adaptation policies in years to come. For mitigation, policy makers need more precise predictions about how much more likely dangerous climate change will occur, as a function of anticipated atmospheric greenhouse-gas concentrations. For adaptation, predictions are needed to guide decisions on infrastructure investment. For example, how will patterns of precipitation change; what parts of the world need to be prepared for water shortages and what parts of the world need to be prepared for more frequent and devastating flooding?

Reducing uncertainty in climate prediction, both global and regional, requires improvements in the models used to predict climate. These models are similar in many respects to the types of weather forecast model discussed above, but differ in two key respects. First, because climate models have to be run over century time scales, rather than days, they must include processes like dynamic sea ice and biogeochemistry, processes that are not especially relevant for weather prediction. This makes the climate models intrinsically more complex than weather prediction models. Owing to this additional complexity and the need to simulate climate on longer time-scales than numerical weather prediction models, climate models typically have much coarser grid resolution than weather prediction models: hundreds of kilometres rather than tens of kilometres.

On the other hand, as with weather prediction, neglecting the small-scale motions causes problems. For climate models, it causes the models to drift compared with reality, even for variables that, in principle, are well resolved in terms of the model's grid spacing. The problem of systematic error is an endemic problem in climate modelling. One of the primary goals of any climate-modelling centre is to eliminate, or at least minimize, this systematic drift. To give one example, many climate models have difficulty simulating the atmospheric phenomenon known as persistent anticyclonic blocking. However, such persistent anticyclonic blocks are the primary cause of drought in many locations; a persistent block causes rain-bearing weather systems to be diverted away from the region of interest. Hence, in order to know whether such a region is likely to be more prone to drought under climate change, it is necessary to know whether the frequency of occurrence of persistent blocking anticyclones will increase in that region as a result of increases in greenhouse-gas concentrations. However, if the models have difficulty simulating the blocking phenomenon in the first place, due to systematic drift, they are not well placed to answer this key question.

Clearly a potential solution to the problem of model drift is to reduce the grid spacing, e.g. to that of contemporary numerical weather prediction models. However, to do this would require computing resources beyond the means of most climate institutes. For example, to run century-long time-scale integrations with a 10 km grid would require multi-petaflop computing capability.

This raises a fundamental theoretical question. How can we expect uncertainty in our predictions of climate change to reduce as the grid spacing reduces? If we look to our knowledge of the mathematical properties of the Navier–Stokes equations for guidance, we are left with a potential dilemma: a simple scaling argument based on the Kolmogorov turbulence suggests that any systematic truncation error, no matter how small scale it may be, can infect the large-scale systematic error of the model in finite time. Whether the Navier–Stokes equations really have this property is the topic of one of the unsolved million-dollar Clay Mathematics Millennium Prize Problems.

This analysis suggests that, effectively, solutions of the dynamic and thermodynamic equations may have some irreducible uncertainty. In this case, it makes sense to try to treat at least the small-scale components of the flow by computationally simple stochastic processes, rather than by the conventional deterministic bulk formula.

This should not be seen as a council of despair, but as a way forward for a problem, climate prediction, that is arguably the most challenging of problems in computational science. For example, let us return to the problem of simulating persistent blocking anticyclones. One way of thinking of the persistent blocking anticyclone is as a preferred regime in the state space of our climate. However, it is secondary to the normal westerly flow that could be viewed as defining the dominant flow regime. Hence think of a double-well potential, the deeper of which represents normal westerly flow, the shallower representing blocking anticyclonic flow. With a highly resolved model, it should be possible not only to represent this potential well but also the right transition frequency between regimes. With a lower resolution model, perhaps the potential well structure is resolved, but the model is sufficiently damped and inactive that the state resides too frequently in the dominant, deeper, westerly flow regime. As a result, this low-resolution model will exhibit a westerly systematic bias, and be poor at simulating spells of persistent anticyclonic weather. However, if this is the case, then injecting stochastic noise into the near-grid scale may be sufficient to lead to a significant improvement in simulating the correct regime statistics.

Hence, as well as exploring the benefits of high resolution (and this work must certainly be done), in addition climate modellers should also explore the benefits of improving the representation of near and subgrid flow in lower resolution models by stochastic processes. In practice, it is quite probable that these pursuits are not mutually exclusive: as explicit resolution approaches that associated with individual convective cloud systems, the unresolved sub-cloud dynamics will then be represented stochastically.

In his study of the economics of climate change, Lord Stern has shown that the climate problem is, globally, a trillion-dollar problem (Stern 2006). Reliable global and regional climate predictions with accurate error bars are an essential element in trying to combat the threat of climate change. This is the reason why, at the beginning of this Introduction, we suggested that the theme of this issue is itself a trillion-dollar theme!

We believe we are at the beginning of a new era in weather and climate modelling—an era that recognizes that although the equations of motion are formally deterministic, the best predictions, whether of weather on time scales of days, or climate on time scales of a century or more, will be based on models that are at least partially stochastic.

2. Contents

This Theme Issue, consisting of 11 invited papers, gathers together the latest research findings in stochastic physics and climate modelling. The first three papers explore the mathematical rationale behind stochastic climate modelling, and the effects in conceptual models. In the first paper, Andrew Majda, Christian Franzke and Boualem Khouider offer an applied mathematics perspective on stochastic modelling for climate (Majda *et al.* 2008). They develop a new low-dimensional stochastic model that mimics key features of atmospheric general circulation models, in order to test the fidelity of stochastic-mode reduction procedures. In the next paper, Cécile Penland and Brian Ewald review the basic properties of stochastic differential equations driven by noise (Penland & Ewald 2008). They also discuss aspects of numerically generating random noise processes. In the third paper, Daniel Wilks discusses the effects of stochastic parametrization in conceptual climate models (Wilks 2008). He notes that, in addition to enhancing the qualitative fidelity to the corresponding real climate system, stochastic parametrization can allow models to exhibit rich new behaviours of which their deterministic counterparts are incapable.

The next four papers apply stochastic techniques to the modelling of turbulence and seasonal, decadal and centennial variability. First, Balasubramanya Nadiga examines the orientation of eddy fluxes in geostrophic turbulence (Nadiga 2008). His findings point to a fundamentally new approach to parametrizing the effects of eddies in the global ocean circulation. Next, Richard Kleeman explores stochastic theories for the irregularity of the El Niño/Southern Oscillation (Kleeman 2008), paying particular attention to explanations that involve stochastic forcing of the slow ocean modes by fast atmospheric transients. Then, Adam Monahan, Julie Alexander and Andrew Weaver examine the time scales and patterns of variability in stochastic models of the ocean's meridional overturning circulation (Monahan *et al.* 2008), including impacts on variability, regime transitions and the dynamics of Dansgaard–Oeschger events. In the next paper, Henk Dijkstra, Leela Frankcombe and Anna von der Heydt present a stochastic dynamical systems view of the Atlantic Multidecadal Oscillation (Dijkstra *et al.* 2008) and suggest that a stochastic Hopf bifurcation is involved in the multidecadal variability of the North Atlantic.

The final four papers consider specific examples of stochastic parametrization schemes in state-of-the-art climate models. First, Judith Berner, Francisco Doblas-Reyes, Tim Palmer, Glen Shutts and Antje Weisheimer analyse the impact of a quasi-stochastic cellular automaton backscatter scheme in a coupled ocean–atmosphere model (Berner *et al.* 2008). They find that systematic errors are significantly reduced, and that the probabilistic skill of seasonal forecasts is significantly improved. Next, J. David Neelin, Ole Peters, Johnny Lin, Katrina Hales and Christopher Holloway present some observational constraints on stochastic convective schemes (Neelin *et al.* 2008) that shed new light on the validity of the decades-old convective quasi-equilibrium assumption. Then, Michael Ball and Robert Plant discuss the potential usefulness of single-column models for testing stochastic physics schemes (Ball & Plant 2008), using simulations of transitions between active and suppressed periods of tropical convection as an illustration. In the final paper, Glenn Shutts, Thomas Allen and Judith Berner speculatively propose extending current stochastic parametrization methods using techniques adopted from the field of computer graphics (Shutts *et al.* 2008). Models used in computer games and visualization software illustrate the potential for cheap but realistic simulations.

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